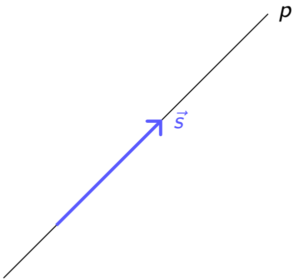
6.3. Jednadžba pravca u prostoru

22. 1. 2020.

Fotografija: Johannes Plenio

Vektor smjera

Neka je $p \subseteq \mathbb{R}^3$ pravac. Njegov **vektor smjera** $\vec{s}$ jest svaki vektor $\vec{s} \in V^3 \setminus \{\vec{0}\}$ takav da je $\vec{s} \parallel p$.



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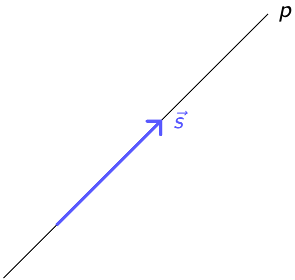
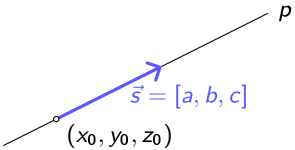
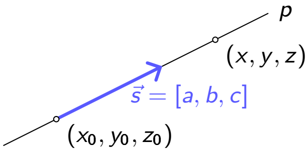


Takav $\vec{s}$ nije jedinstven! Jedinstven mu je samo smjer.

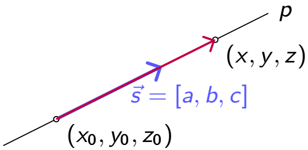
Parametarski oblik jednadžbe pravca



Parametarski oblik jednadžbe pravca



Parametarski oblik jednadžbe pravca



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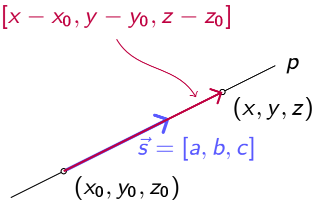

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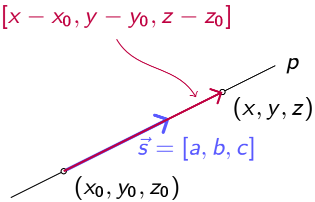
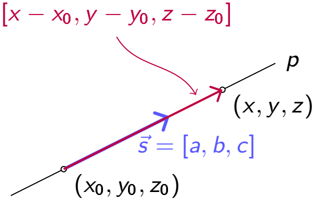


Pravac p kroz točku (x_0, y_0, z_0) s vektorom smjera $\vec{s} = [a, b, c]$ čine točke $(x, y, z) \in \mathbb{R}^3$ sa svojstvom

$$[x - x_0, y - y_0, z - z_0] \parallel \vec{s},$$

Parametarski oblik jednadžbe pravca



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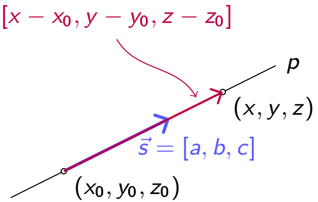
$$[x - x_0, y - y_0, z - z_0] \parallel \vec{s},$$

tj.

$$[x - x_0, y - y_0, z - z_0] = t[a, b, c] \quad \text{za neki } t \in \mathbb{R},$$

Parametarski oblik jednadžbe pravca



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$$[x - x_0, y - y_0, z - z_0] \parallel \vec{s},$$

tj.

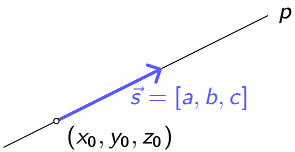
$$[x - x_0, y - y_0, z - z_0] = t[a, b, c] \quad \text{za neki } t \in \mathbb{R},$$

dakle

$$\boxed{p \ldots \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \quad \text{za neki } t \in \mathbb{R}}$$

jest jednadžba pravca p, tzv. **parametarski oblik jednadžbe pravca** p.

Parametarski oblik jednadžbe pravca



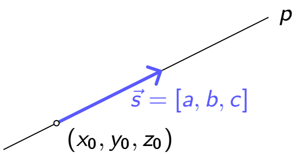
Važna napomena. Pravac p je dan jednadžbom

$$\boxed{p \ldots \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \quad \text{za neki } t \in \mathbb{R}}$$

ako i samo ako vrijede sljedeći uvjeti:

- Točka $(x_0, y_0, z_0) \in \mathbb{R}^3$ leži na pravcu p.
- Vektor $[a, b, c] \in V^3 \setminus \{\vec{0}\}$ je vektor smjera pravca p.

Kanonski oblik jednadžbe pravca



Uobičajeno je jednadžbu

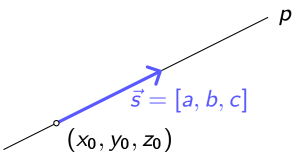
$$\boxed{p \ldots \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \quad \text{za neki } t \in \mathbb{R}}$$

kraće zapisivati u obliku

$$\boxed{p \ldots \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c},}$$

koji se zove **kanonski oblik jednadžbe pravca** p.

Kanonski oblik jednadžbe pravca



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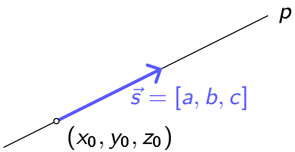
kraće zapisivati u obliku

$$\boxed{p \ldots \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c},}$$

koji se zove **kanonski oblik jednadžbe pravca** p. (Moguće je $a = 0$ ili $b = 0$ ili $c = 0$, samo ne $a = b = c = 0$.)

Kanonski oblik jednadžbe pravca



Važna napomena. Pravac p dan je jednadžbom

$$\boxed{p \ldots \ \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}}$$

ako i samo ako vrijede sljedeći uvjeti:

- Točka $(x_0, y_0, z_0) \in \mathbb{R}^3$ leži na pravcu p.
- Vektor $[a, b, c] \in V^3 \setminus \{\vec{0}\}$ je vektor smjera pravca p.

Napomena. Kanonski i parametarski oblik jednadžbe pravca p nisu jedinstveni: vektor $[a, b, c]$ je jedinstven samo do na množenje skalarom iz $\mathbb{R} \setminus \{0\}$, dok (x_0, y_0, z_0) može biti bilo koja točka pravca p.

Zadatak 62

Napišite jednadžbu pravca p kroz točke $A = (1, 1, 1)$ i $B = (1, 0, -1)$.

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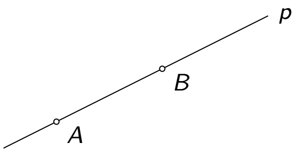
Rješenje.



Zadatak 62

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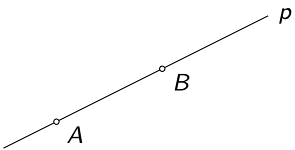

Sjetimo se: pravac p s vektorom smjera $\vec{s} = [a, b, c]$ kroz točku (x_0, y_0, z_0) dan je jednadžbom

$$\boxed{p \ldots \ \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.}$$

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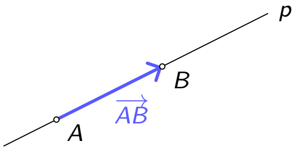

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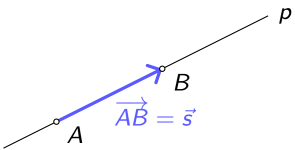

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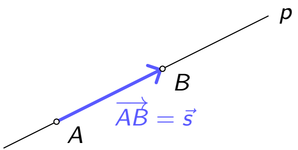

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$$\boxed{p \ldots \ \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.}$$

- Očito je $\overrightarrow{AB}$ vektor smjera od p pa možemo staviti $[a, b, c] = \overrightarrow{AB} = [1 - 1, 0 - 1, -1 - 1] = [0, -1, -2]$.

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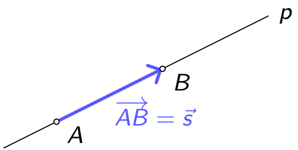

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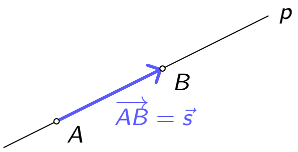
- Kako je $A \in p$, možemo staviti $(x_0, y_0, z_0) = A = (1, 1, 1)$.

Zadatak 62

Napišite jednadžbu pravca p kroz točke $A = (1,1,1)$ i $B = (1,0,-1)$.

Rješenje.



Sjetimo se: pravac p s vektorom smjera $\vec{s} = [a, b, c]$ kroz točku (x_0, y_0, z_0) dan je jednadžbom

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- Očito je $\overrightarrow{AB}$ vektor smjera od p pa možemo staviti $[a, b, c] = \overrightarrow{AB} = [1 - 1, 0 - 1, -1 - 1] = [0, -1, -2]$.
- Kako je $A \in p$, možemo staviti $(x_0, y_0, z_0) = A = (1, 1, 1)$.

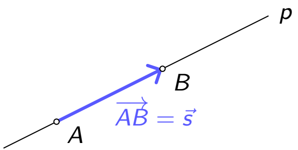
Dakle, pravac p je dan jednadžbom

$$p \ldots \ \frac{x - 1}{0} = \frac{y - 1}{-1} = \frac{z - 1}{-2}.$$

Zadatak 62

Napišite jednadžbu pravca p kroz točke $A = (1,1,1)$ i $B = (1,0,-1)$.

Rješenje.



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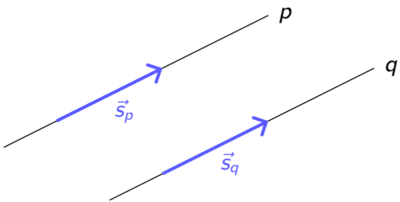
- Očito je $\overrightarrow{AB}$ vektor smjera od p pa možemo staviti $[a,b,c] = \overrightarrow{AB} = [1-1, 0-1, -1-1] = [0,-1,-2]$.
- Kako je $A \in p$, možemo staviti $(x_0, y_0, z_0) = A = (1,1,1)$.

Dakle, pravac p je dan jednadžbom

$$p \ldots \ \frac{x-1}{0} = \frac{y-1}{-1} = \frac{z-1}{-2}.$$

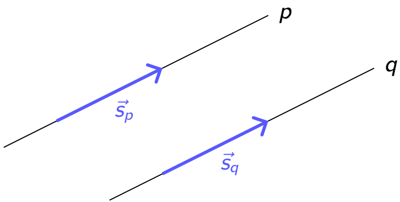
Napomena. Parametarski oblik jednadžbe pravca p: $p \ldots \begin{cases} x = 1 \\ y = 1 - t \\ z = 1 - 2t. \end{cases}$

Paralelnost i okomitost pravaca



$$\boxed{p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q}$$

Paralelnost i okomitost pravaca



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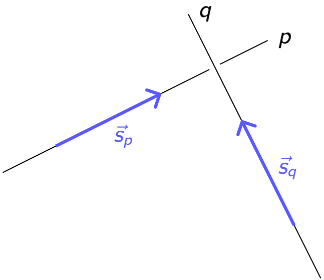

$$\boxed{p \perp q \quad \Leftrightarrow \quad \vec{s}_p \perp \vec{s}_q}$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \ \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Zadatak 63

Odredite odnos pravaca

$$p \ldots \ \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} \ :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \ \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

Zadatak 63

Odredite odnos pravaca

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Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.

Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.

Sijeku li se p i q?

Zadatak 63

Odredite odnos pravaca

$$p \dots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \dots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} \, :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.

Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \left\{ \right.$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \not\parallel q$.

Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t \quad \text{za neki } t \in \mathbb{R} \\ \end{cases}$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.

Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \ \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

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dakle $p \nparallel q$.

Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x = & t-1 \\ y = & t \\ z = & 2t+1 \end{cases}$$

Zadatak 63

Odredite odnos pravaca

$$p \dots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \dots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} \,:$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

Rješenje. Imamo

$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.
Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x = t - 1 = s \\ y = t \\ z = 2t + 1 \end{cases}$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \ \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} \ :$$

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$$\Leftrightarrow \quad \begin{cases} x = & t - 1 = s \\ y = & t = 3s - 1 \\ z = & 2t + 1 \end{cases}$$

Zadatak 63

Odredite odnos pravaca

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$$\Leftrightarrow \quad \begin{cases} x = \; t - 1 = s \\ y = \; t = 3s - 1 \\ z = \; 2t + 1 = 4s + 2 \end{cases} \qquad \text{za neke } s, t \in \mathbb{R}.$$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

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$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

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Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

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$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

$$\Leftrightarrow \quad \left\{ \begin{array}{l} x = \\ y = \\ z = \end{array} \boxed{\begin{array}{l} t-1 = s \\ t = 3s-1 \\ 2t+1 = 4s+2 \end{array}} \right. \qquad \text{za neke } s,t \in \mathbb{R}.$$

Kako uokvireni sustav nema rješenja, ne postoje takvi $s,t \in \mathbb{R}$, dakle ne postoji nijedna točka $(x,y,z) \in p \cap q$

Zadatak 63

Odredite odnos pravaca

$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \quad \text{i} \quad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4}:$$

jesu li paralelni, mimoilazni (nisu paralelni i ne sijeku se) ili se sijeku u jednoj točki.

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$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

dakle $p \nparallel q$.

Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

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Kako uokvireni sustav nema rješenja, ne postoje takvi $s,t \in \mathbb{R}$, dakle ne postoji nijedna točka $(x,y,z) \in p \cap q$, tj. $p \cap q = \emptyset$.

Zadatak 63

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$$p \ldots \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} \qquad \text{i} \qquad q \ldots \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} :$$

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$$p \parallel q \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{s}_q \quad \Leftrightarrow \quad [1,1,2] \parallel [1,3,4],$$

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Sijeku li se p i q? Za $(x,y,z) \in \mathbb{R}^3$ vrijedi

$$(x,y,z) \in p \cap q \quad \Leftrightarrow \quad \begin{cases} \frac{x+1}{1} = \frac{y}{1} = \frac{z-1}{2} = t & \text{za neki } t \in \mathbb{R} \\ \frac{x}{1} = \frac{y+1}{3} = \frac{z-2}{4} = s & \text{za neki } s \in \mathbb{R} \end{cases}$$

$$\Leftrightarrow \quad \left\{ \begin{array}{l} x = \\ y = \\ z = \end{array} \right. \boxed{\begin{array}{l} t - 1 = s \\ t = 3s - 1 \\ 2t + 1 = 4s + 2 \end{array}} \qquad \text{za neke } s, t \in \mathbb{R}.$$

Kako uokvireni sustav nema rješenja, ne postoje takvi $s, t \in \mathbb{R}$, dakle ne postoji nijedna točka $(x,y,z) \in p \cap q$, tj. $p \cap q = \emptyset$. Dakle, pravci p i q su mimoilazni.

Zadatak 64

Odredite sjecište pravca

$$p \dots \ \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

i ravnine

$$\pi \dots \ x - y + 4z = 5.$$

Zadatak 64

Odredite sjecište pravca

$$p \ldots \; \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

i ravnine

$$\pi \ldots \; x - y + 4z = 5.$$

Rješenje. Imamo

$$(x, y, z) \in p \cap \pi \quad \Leftrightarrow \quad \left\{ \vphantom{\begin{array}{c} a \\ b \\ c \\ d \end{array}} \right.$$

Zadatak 64

Odredite sjecište pravca

$$p \ldots \ \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

i ravnine

$$\pi \ldots \ x - y + 4z = 5.$$

Rješenje. Imamo

$$(x, y, z) \in p \cap \pi \quad \Leftrightarrow \quad \begin{cases} x = 2t + 1 \\ y = t \\ z = 2t - 2 \\ \end{cases} \qquad \text{za neki } t \in \mathbb{R}$$

Zadatak 64

Odredite sjecište pravca

$$p \ldots \ \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

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$$(x,y,z) \in p \cap \pi \quad \Leftrightarrow \quad \begin{cases} x = 2t+1 \\ y = t \\ z = 2t-2 \\ x - y + 4z = 5 \end{cases} \qquad \text{za neki } t \in \mathbb{R}$$

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$$\Leftrightarrow \quad \begin{cases} x = 2t+1 \\ y = t \\ z = 2t-2 \\ (2t+1) - t + 4(2t-2) = 5 \end{cases} \quad \text{za neki } t \in \mathbb{R}$$

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Odredite sjecište pravca

$$p \ldots \ \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

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Zadatak 64

Odredite sjecište pravca

$$p \ldots \ \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

i ravnine

$$\pi \ldots \ x - y + 4z = 5.$$

Rješenje. Imamo

$$(x,y,z) \in p \cap \pi \quad \Leftrightarrow \quad \begin{cases} x = 2t+1 \\ y = t \\ z = 2t-2 \\ x - y + 4z = 5 \end{cases} \quad \text{za neki } t \in \mathbb{R}$$

$$\Leftrightarrow \quad \begin{cases} x = 2t+1 \\ y = t \\ z = 2t-2 \\ (2t+1) - t + 4(2t-2) = 5 \end{cases} \quad \text{za neki } t \in \mathbb{R} \quad \Leftrightarrow \quad \begin{cases} x = \frac{11}{3} \\ y = \frac{4}{3} \\ z = \frac{2}{3} \\ t = \frac{4}{3}. \end{cases}$$

Zadatak 64

Odredite sjecište pravca

$$p \ldots \; \frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{2}$$

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Rješenje. Imamo

$$(x,y,z) \in p \cap \pi \quad \Leftrightarrow \quad \begin{cases} x = 2t+1 \\ y = t \\ z = 2t-2 \\ x - y + 4z = 5 \end{cases} \quad \text{za neki } t \in \mathbb{R}$$

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Dakle, traženo sjecište je točka $\left(\frac{11}{3}, \frac{4}{3}, \frac{2}{3}\right)$.

Zadavanje pravca

Tri najčešća načina zadavanja pravca p:

Zadavanje pravca

Tri najčešća načina zadavanja pravca p:

- jednom točkom i vektorom smjera ($\rightsquigarrow$ parametarski oblik jednadžbe pravca)

Zadavanje pravca

Tri najčešća načina zadavanja pravca p:

- jednom točkom i vektorom smjera ($\rightsquigarrow$ parametarski oblik jednadžbe pravca)
- dvjema točkama (vidi Zadatak 62)

Zadavanje pravca

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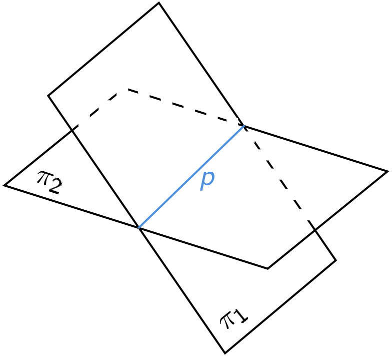
- jednom točkom i vektorom smjera ($\rightsquigarrow$ parametarski oblik jednadžbe pravca)
- dvjema točkama (vidi Zadatak 62)
- kao presjek dviju ravnina $\pi_1 \nparallel \pi_2$:



Zadavanje pravca

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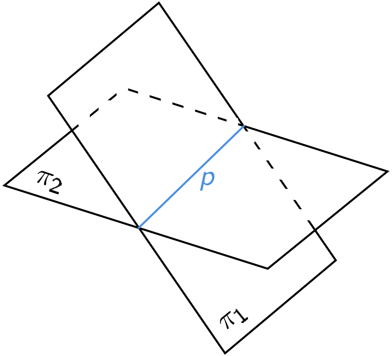
- jednom točkom i vektorom smjera ($\rightsquigarrow$ parametarski oblik jednadžbe pravca)
- dvjema točkama (vidi Zadatak 62)
- kao presjek dviju ravnina $\pi_1 \nparallel \pi_2$:

$$\boxed{p \ldots \begin{cases} \pi_1 \ldots \ A_1x + B_1y + C_1z = D_1 \\ \pi_2 \ldots \ A_2x + B_2y + C_2z = D_2 \end{cases}}$$



– **implicitni oblik jednadžbe pravca** p.

Tri najčešća načina zadavanja pravca p:

- jednom točkom i vektorom smjera ($\rightsquigarrow$ parametarski oblik jednadžbe pravca)
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- kao presjek dviju ravnina $\pi_1 \nparallel \pi_2$:

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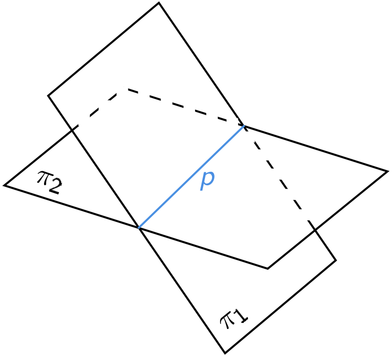
– **implicitni oblik jednadžbe pravca** p.

Napomena. Implicitni oblik jednadžbe pravca p očito nije jedinstven (pravac p može se prikazati kao presjek dviju ravnina na beskonačno mnogo načina).



Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

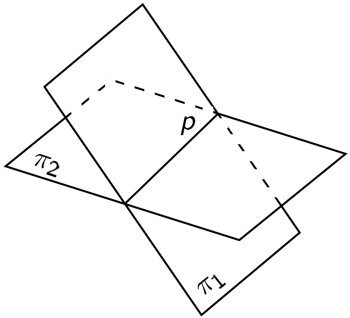
Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \dots \begin{cases} x+y+z=1 & \dots \pi_1 \\ x-y+z=1. & \dots \pi_2 \end{cases}$

Rješenje. 1. način.

Zadatak 65

Odredite kanonski oblik jednadžbe pravca p ... $\begin{cases} x+y+z=1 & \ldots \pi_1 \\ x-y+z=1. & \ldots \pi_2 \end{cases}$



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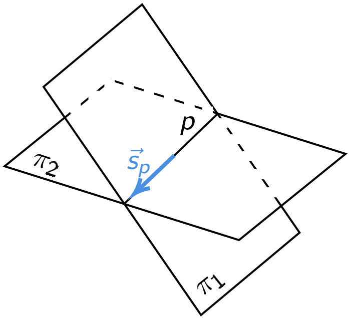

Zadatak 65

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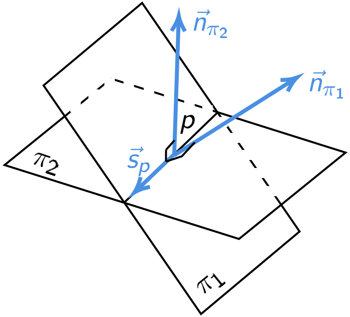

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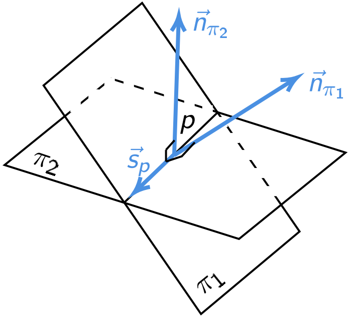


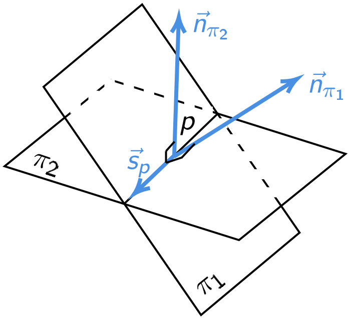
Rješenje. 1. način.

- Kako je $\vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$, možemo staviti

$$\vec{s}_p = \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} =$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p\ldots$ $\begin{cases} x+y+z=1 & \ldots \pi_1 \\ x-y+z=1. & \ldots \pi_2 \end{cases}$



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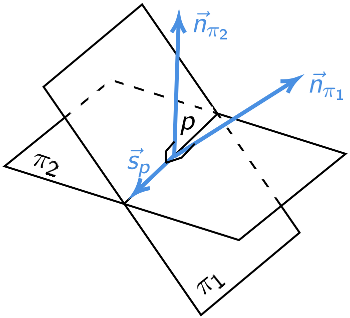
- Kako je $\vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$, možemo staviti

$$\vec{s}_p = \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x + y + z = 1 & \ldots \pi_1 \\ x - y + z = 1. & \ldots \pi_2 \end{cases}$



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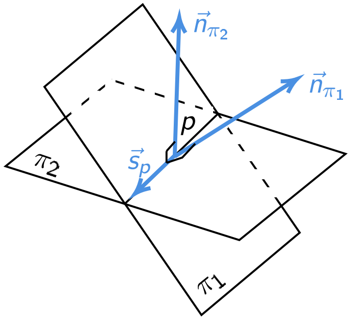
Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 & \ldots \pi_1 \\ x-y+z=1. & \ldots \pi_2 \end{cases}$



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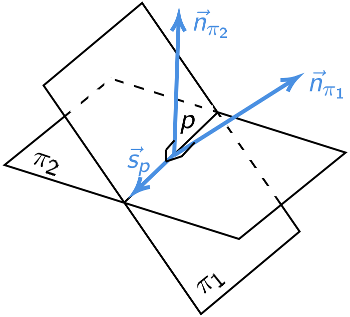
- Npr. pogodimo da je $T := (0,0,1) \in p$ (jer zadovoljava njegovu jednadžbu).

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 & \ldots \pi_1 \\ x-y+z=1. & \ldots \pi_2 \end{cases}$



Rješenje. 1. način.

- Kako je $\vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$, možemo staviti

$$\vec{s}_p = \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = [2, 0, -2].$$

- Npr. pogodimo da je $T := (0, 0, 1) \in p$ (jer zadovoljava njegovu jednadžbu).

Sad uvrštavanjem $[a, b, c] = \vec{s}_p = [2, 0, -2]$ i $(x_0, y_0, z_0) = T = (0, 0, 1)$ u formulu

$$p \ldots \quad \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} \qquad \text{dobivamo} \qquad p \ldots \quad \frac{x}{2} = \frac{y}{0} = \frac{z-1}{-2}.$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x + y + z = 1 \\ x - y + z = 1. \end{cases}$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} \\ y=0 \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x+z=1 \\ y=0 \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x+z=1 \\ y=0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x=1-z \\ y=0 \\ \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z)\in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x+z=1 \\ y=0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x=1-z \\ y=0 \\ z=t \text{ za neki } t\in\mathbb{R} \end{cases}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z)\in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x+z=1 \\ y=0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x=1-z \\ y=0 \\ z=t \text{ za neki } t\in\mathbb{R} \end{cases} \quad \Leftrightarrow \quad \boxed{\begin{cases} x=1-t \\ y=0 \\ z=t \end{cases} \quad \text{za neki } t\in\mathbb{R}.}$$

Zadatak 65

Odredite kanonski oblik jednadžbe pravca $p \ldots \begin{cases} x+y+z=1 \\ x-y+z=1. \end{cases}$

Rješenje. 2. način. Imamo

$$(x,y,z) \in p \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ x-y+z=1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} x+y+z=1 & (1) \\ 2y=0 & (1)-(2) \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x+z=1 \\ y=0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x=1-z \\ y=0 \\ z=t \text{ za neki } t \in \mathbb{R} \end{cases} \quad \Leftrightarrow \quad \boxed{\begin{cases} x=1-t \\ y=0 \\ z=t \end{cases} \quad \text{za neki } t \in \mathbb{R}.}$$

Uokvirena je jednadžba parametarski oblik jednadžbe pravca p, pa je kanonski oblik dan sa

$$p \ldots \frac{x-1}{-1} = \frac{y}{0} = \frac{z}{1}.$$

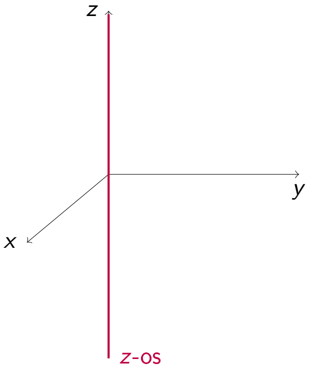
Zadatak 66

Odredite jednadžbu z-osi.

Zadatak 66

Odredite jednadžbu z-osi.

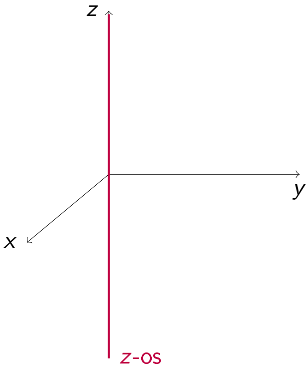
Rješenje. 1. način.



Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 1. način.

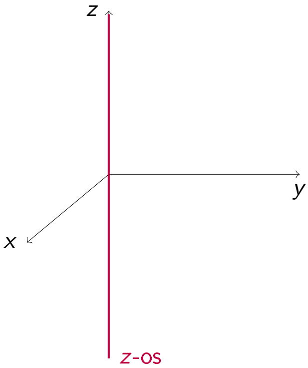


z-os očito čine točke $(x, y, z) \in \mathbb{R}^3$ kojima su x- i y-koordinata 0.

Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 1. način.



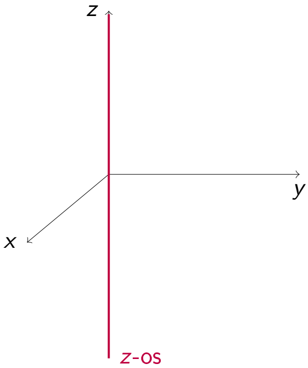
z-os očito čine točke $(x, y, z) \in \mathbb{R}^3$ kojima su x- i y-koordinata 0. Dakle, implicitni oblik jednadžbe z-osi dan je sa

$$z\text{-os} \ldots \begin{cases} x = 0 \\ y = 0. \end{cases}$$

Zadatak 66

Odredite jednadžbu z-osi.

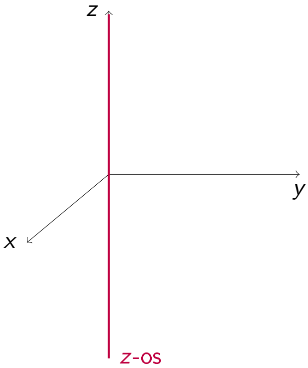
Rješenje. *2. način.*



Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 2. način.



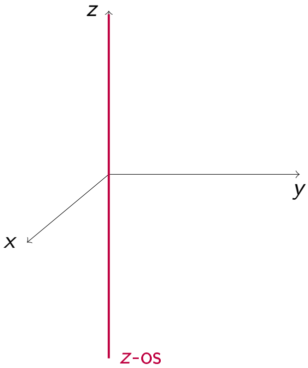
z-os očito čine točke $(x, y, z) \in \mathbb{R}^3$ kojima su x- i y-koordinata 0, dakle

$$z\text{-os} = \{(0, 0, t) : t \in \mathbb{R}\} .$$

Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 2. način.



z-os očito čine točke $(x, y, z) \in \mathbb{R}^3$ kojima su x- i y-koordinata 0, dakle

$$z\text{-os} = \{(0, 0, t) : t \in \mathbb{R}\}.$$

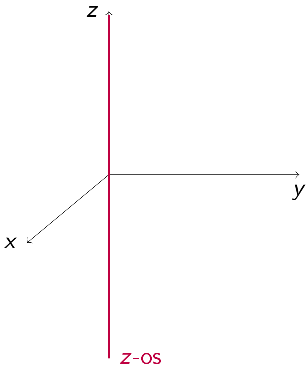
Prema tome, parametarski oblik jednadžbe z-osi je

$$z\text{-os} \ldots \begin{cases} x = 0 \\ y = 0 \\ z = t \end{cases} \quad \text{za neki } t \in \mathbb{R}.$$

Zadatak 66

Odredite jednadžbu z-osi.

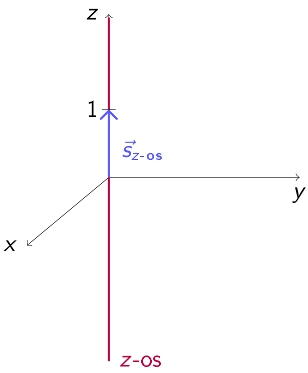
Rješenje. *3. način.*



Zadatak 66

Odredite jednadžbu z-osi.

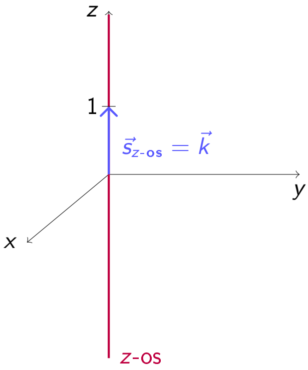
Rješenje. 3. način.



Zadatak 66

Odredite jednadžbu z-osi.

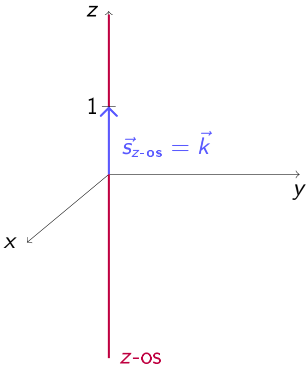
Rješenje. 3. način.



Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 3. način.



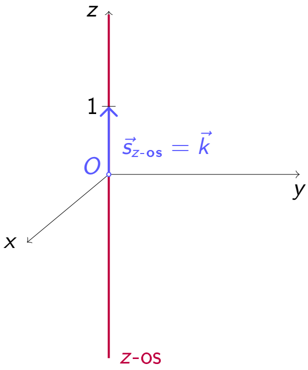
Očito vrijedi:

- $\vec{k} = [0, 0, 1]$ je vektor smjera z-osi.

Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 3. način.



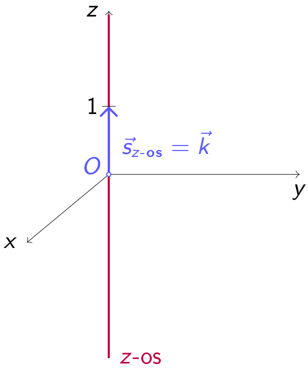
Očito vrijedi:

- $\vec{k} = [0, 0, 1]$ je vektor smjera z-osi.

Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 3. način.



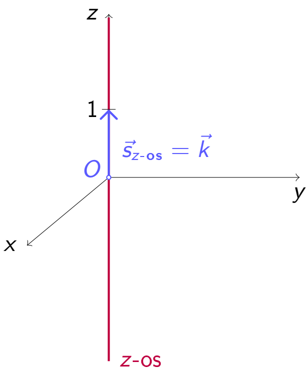
Očito vrijedi:

- $\vec{k} = [0, 0, 1]$ je vektor smjera z-osi.
- $O = (0, 0, 0) \in z$-os.

Zadatak 66

Odredite jednadžbu z-osi.

Rješenje. 3. način.



Očito vrijedi:

- $\vec{k} = [0, 0, 1]$ je vektor smjera z-osi.
- $O = (0, 0, 0) \in z$-os.

Dakle, uvrštavanjem $[a, b, c] = \vec{k} = [0, 0, 1]$ i $(x_0, y_0, z_0) = O = (0, 0, 0)$ u formulu

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

dobivamo da je kanonski oblik jednadžbe z-osi

$$z\text{-os} \ldots \ \frac{x}{0} = \frac{y}{0} = \frac{z}{1}.$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \dots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

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$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ \\ \\ \end{cases}$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ \\ \\ \end{cases}$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ \end{cases}$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ D = 1. \end{cases}$$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ D = 1. \end{cases}$$

Dakle, pravac p siječe z-os ako i samo ako je $D = 1$

Zadatak 67

Odredite $D \in \mathbb{R}$ za koji pravac

$$p \ldots \begin{cases} x - y + z = -1 \\ 2x - 3y - z = D \end{cases}$$

siječe z-os.

Rješenje. U zadatku 66 vidjeli smo da je implicitna jednadžba z-osi $\begin{cases} x = 0 \\ y = 0. \end{cases}$

Prema tome, za $(x, y, z) \in \mathbb{R}^3$ vrijedi

$$(x, y, z) \in p \cap z\text{-os} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ x - y + z = -1 \\ 2x - 3y - z = D \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 0 \\ y = 0 \\ z = -1 \\ D = 1. \end{cases}$$

Dakle, pravac p siječe z-os ako i samo ako je $D = 1$ (i u tom je slučaju sjecište točka $(0, 0, -1)$).

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0, \ \ \pi_2 \ldots 3x - y - z = -2, \ \ \pi_3 \ldots 4x - y - 2z = \lambda$

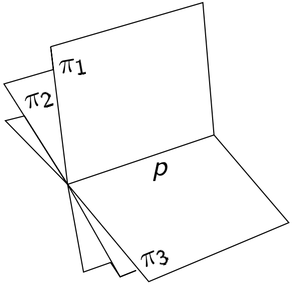
sijeku po istom pravcu.

Zadatak 68

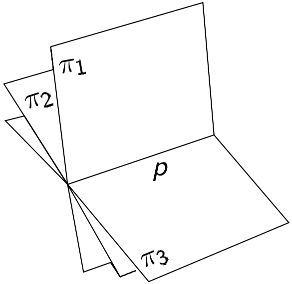
Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Zadatak 68



Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.

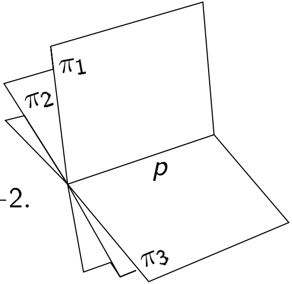
Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



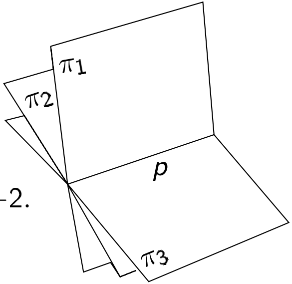
Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0, \;\; \pi_2 \ldots 3x - y - z = -2, \;\; \pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

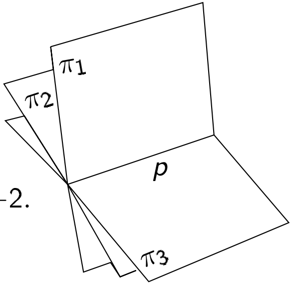
$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \dots x - y + z = 0, \;\; \pi_2 \dots 3x - y - z = -2, \;\; \pi_3 \dots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

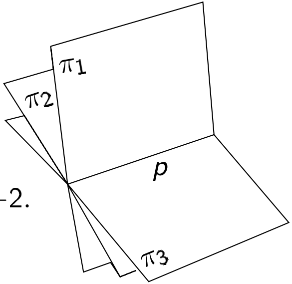
$$p \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0,\ \ \pi_2 \ldots 3x - y - z = -2,\ \ \pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

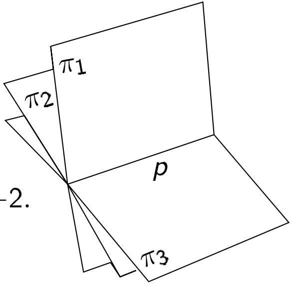
$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

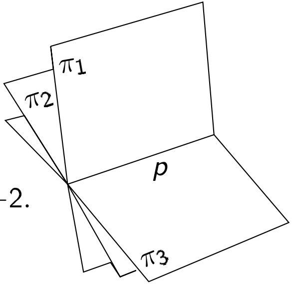
$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0,\ \ \pi_2 \ldots 3x - y - z = -2,\ \ \pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

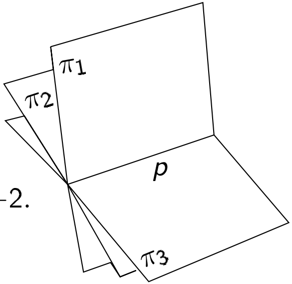
$$\Leftrightarrow \quad \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0,\ \ \pi_2 \ldots 3x - y - z = -2,\ \ \pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

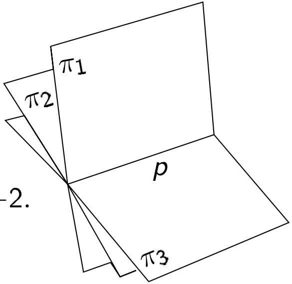
$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \dots x - y + z = 0,\ \ \pi_2 \dots 3x - y - z = -2,\ \ \pi_3 \dots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

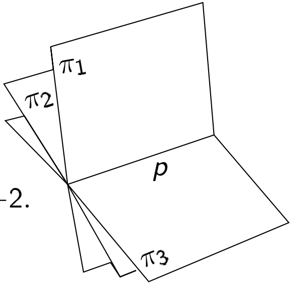
$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t \end{cases} \text{za neki } t \in \mathbb{R}.$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \dots x - y + z = 0, \ \ \pi_2 \dots 3x - y - z = -2, \ \ \pi_3 \dots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t \end{cases} \text{ za neki } t \in \mathbb{R}.$$

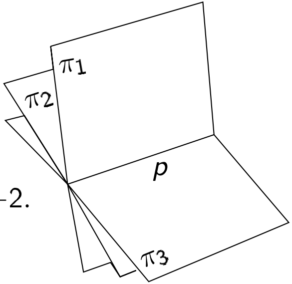
Dakle, pravac p čine točke $(t - 1, 2t - 1, t)$, $t \in \mathbb{R}$.

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t \end{cases} \text{za neki } t \in \mathbb{R}.$$

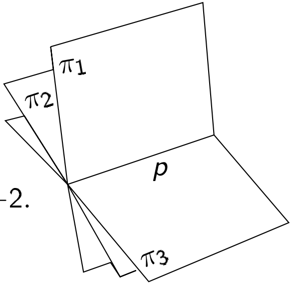
Dakle, pravac p čine točke $(t - 1, 2t - 1, t)$, $t \in \mathbb{R}$. Tražimo $\lambda \in \mathbb{R}$ za koji su sve one u ravnini π_3

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \dots x - y + z = 0, \;\; \pi_2 \dots 3x - y - z = -2, \;\; \pi_3 \dots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \dots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t \end{cases} \text{za neki } t \in \mathbb{R}.$$

Dakle, pravac p čine točke $(t-1, 2t-1, t)$, $t \in \mathbb{R}$. Tražimo $\lambda \in \mathbb{R}$ za koji su sve one u ravnini π_3, tj. $\lambda \in \mathbb{R}$ takav da je

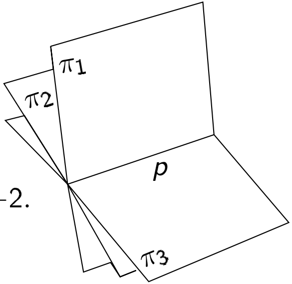
$$4(t-1) - (2t-1) - 2t = \lambda \quad \text{za sve } t \in \mathbb{R}$$

Zadatak 68

Odredite $\lambda \in \mathbb{R}$ takav da se ravnine

$\pi_1 \ldots x - y + z = 0$, $\pi_2 \ldots 3x - y - z = -2$, $\pi_3 \ldots 4x - y - 2z = \lambda$

sijeku po istom pravcu.



Rješenje. Taj pravac mora biti pravac $p := \pi_1 \cap \pi_2 \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2. \end{cases}$

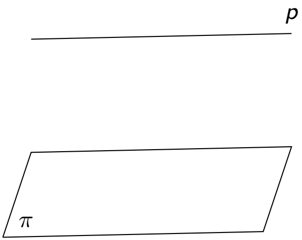
Odredimo parametarski oblik njegove jednadžbe. Imamo

$$p \ldots \begin{cases} x - y + z = 0 \\ 3x - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ 3(y - z) - y - z = -2 \end{cases} \Leftrightarrow \begin{cases} x = y - z \\ y = 2z - 1 \end{cases}$$

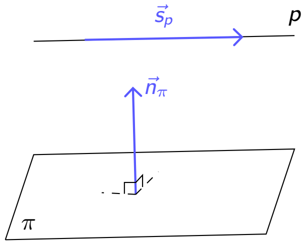
$$\Leftrightarrow \begin{cases} x = (2z - 1) - z \\ y = 2z - 1 \end{cases} \Leftrightarrow \begin{cases} x = z - 1 \\ y = 2z - 1 \\ z = t \text{ za neki } t \in \mathbb{R} \end{cases} \Leftrightarrow \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t \end{cases} \text{ za neki } t \in \mathbb{R}.$$

Dakle, pravac p čine točke $(t-1, 2t-1, t)$, $t \in \mathbb{R}$. Tražimo $\lambda \in \mathbb{R}$ za koji su sve one u ravnini π_3, tj. $\lambda \in \mathbb{R}$ takav da je $4(t-1) - (2t-1) - 2t = \lambda$ za sve $t \in \mathbb{R}$, dakle $\lambda = -3$.

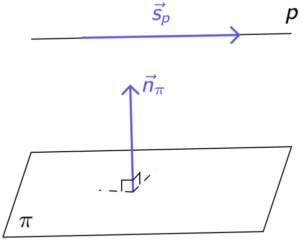
Paralelnost i okomitost pravca i ravnine



Paralelnost i okomitost pravca i ravnine

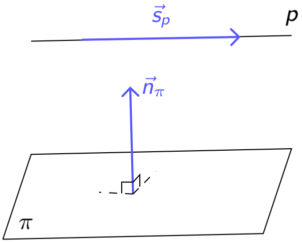


Paralelnost i okomitost pravca i ravnine

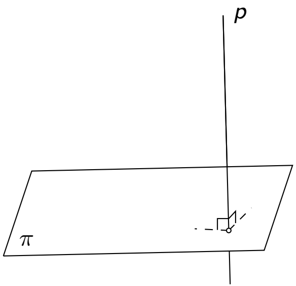


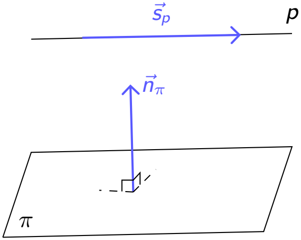
$$\boxed{p \parallel \pi \quad \Leftrightarrow \quad \vec{s}_p \perp \vec{n}_\pi}$$

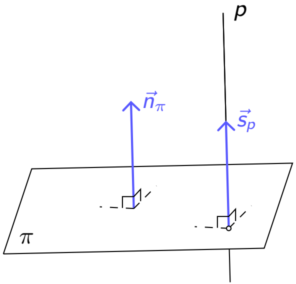
Paralelnost i okomitost pravca i ravnine



$$\boxed{p \parallel \pi \quad \Leftrightarrow \quad \vec{s}_p \perp \vec{n}_\pi}$$

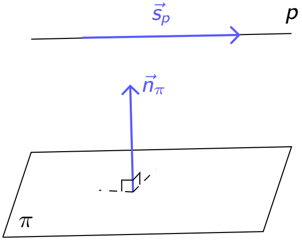




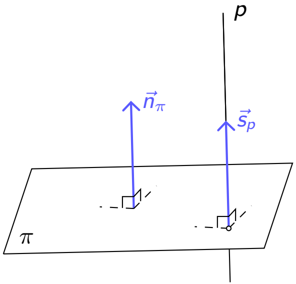


$$\boxed{p \parallel \pi \quad \Leftrightarrow \quad \vec{s}_p \perp \vec{n}_\pi}$$

Paralelnost i okomitost pravca i ravnine



$$p \parallel \pi \quad \Leftrightarrow \quad \vec{s}_p \perp \vec{n}_\pi$$



$$p \perp \pi \quad \Leftrightarrow \quad \vec{s}_p \parallel \vec{n}_\pi$$

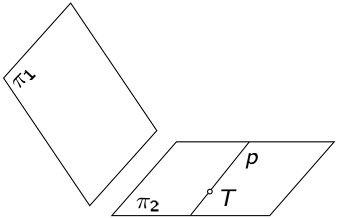
Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

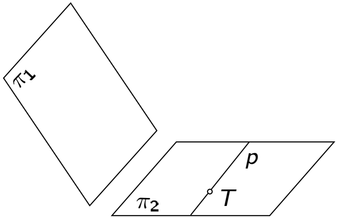
Rješenje.



Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.

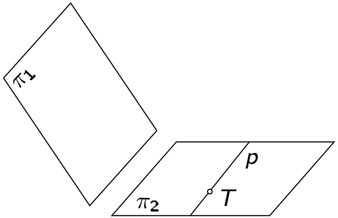


- $p \parallel \pi_1, \pi_2$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x+2y+3z=8$, leži u ravnini $\pi_2 \ldots\ 2x-y+z=3$ i prolazi točkom $T=(1,2,3)$.

Rješenje.

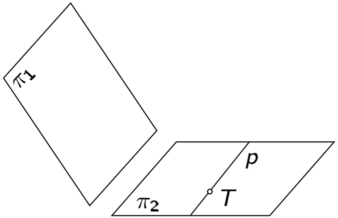


- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



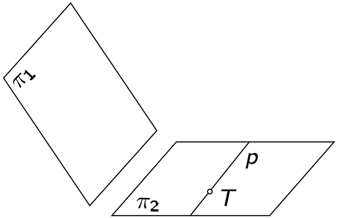
- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2}$$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



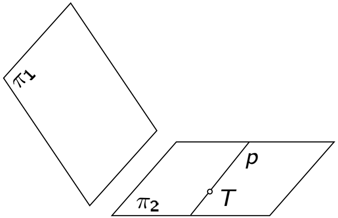
- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix}$$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots \; x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots \; 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



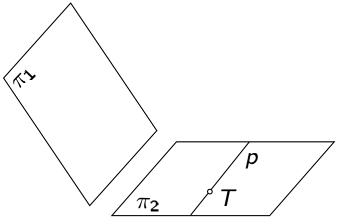
- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = [5, 5, -5]$$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots \; x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots \; 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

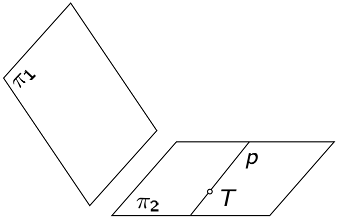
$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = [5, 5, -5]$$

pa možemo staviti npr. $\vec{s}_p = \frac{1}{5}[5, 5, -5]$

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

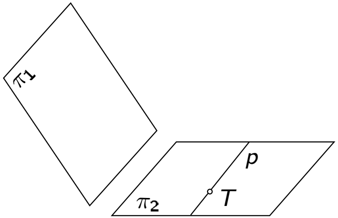
$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = [5, 5, -5]$$

pa možemo staviti npr. $\vec{s}_p = \frac{1}{5}[5, 5, -5] = [1, 1, -1]$.

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x+2y+3z=8$, leži u ravnini $\pi_2 \ldots\ 2x-y+z=3$ i prolazi točkom $T=(1,2,3)$.

Rješenje.



- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

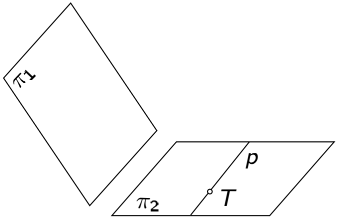
  $$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = [5,5,-5]$$

  pa možemo staviti npr. $\vec{s}_p = \frac{1}{5}[5,5,-5] = [1,1,-1]$.
- $T=(1,2,3) \in p$.

Zadatak 69

Napišite kanonski oblik jednadžbe pravca p koji je paralelan s ravninom $\pi_1 \ldots\ x + 2y + 3z = 8$, leži u ravnini $\pi_2 \ldots\ 2x - y + z = 3$ i prolazi točkom $T = (1, 2, 3)$.

Rješenje.



- $p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$

$$\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix} = [5, 5, -5]$$

  pa možemo staviti npr. $\vec{s}_p = \frac{1}{5}[5, 5, -5] = [1, 1, -1]$.

- $T = (1, 2, 3) \in p$.

Sad uvrštavanjem $[a, b, c] = \vec{s}_p = [1, 1, -1]$ i $(x_0, y_0, z_0) = T = (1, 2, 3)$ u formulu

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

dobivamo da je kanonski oblik jednadžbe pravca p dan sa

$$p \ldots \ \frac{x - 1}{1} = \frac{y - 2}{1} = \frac{z - 3}{-1}.$$

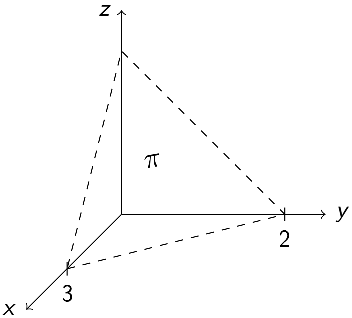
Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

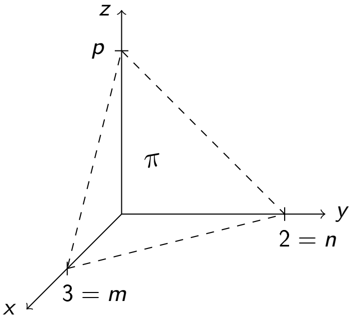
Rješenje.



Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

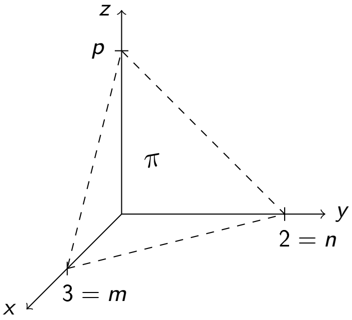
Rješenje.



Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

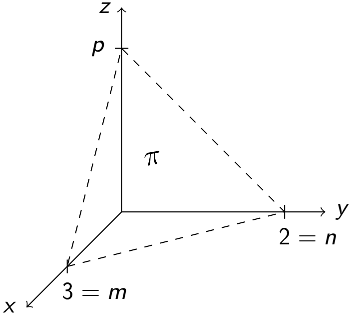
$$\pi \ldots \; \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

za neki $p \in \mathbb{R} \setminus \{0\}$.

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

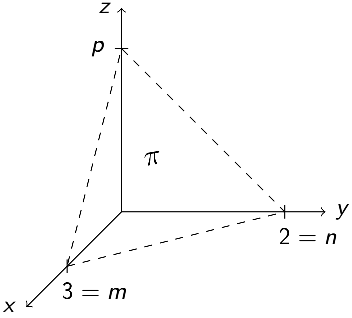
$$\pi \ldots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p.

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \dots \quad \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

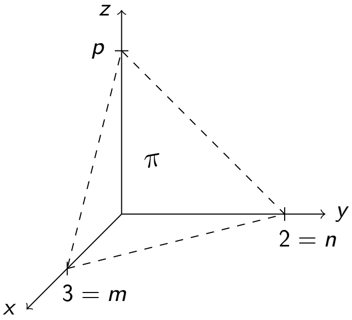
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\pi \parallel \vec{v}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

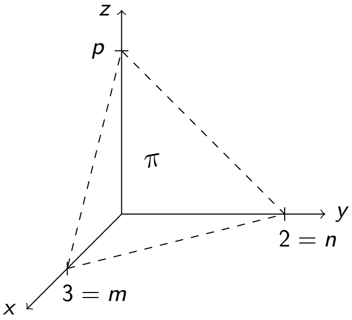
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\pi \parallel \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \perp \vec{v}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \dots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

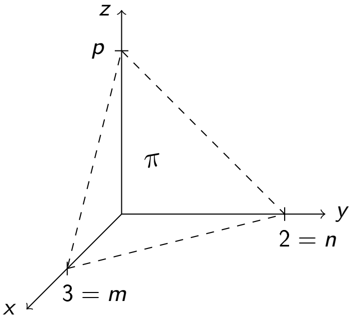
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\pi \parallel \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\frac{x}{m} + \frac{y}{n} + \frac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

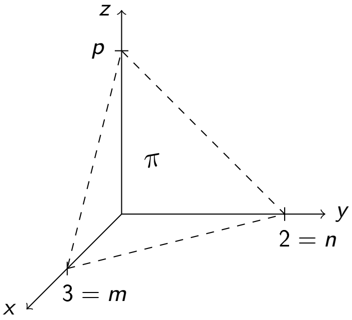
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned} \pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\ &\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \end{aligned}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\frac{x}{m} + \frac{y}{n} + \frac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \quad \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

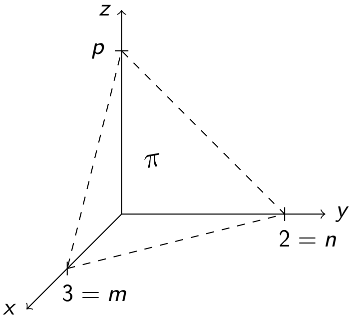
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned} \pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\ &\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \\ &\Leftrightarrow \quad \tfrac{1}{3} \cdot 2 + \tfrac{1}{2} \cdot (-1) + \tfrac{1}{p} \cdot 1 = 0 \end{aligned}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \; \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

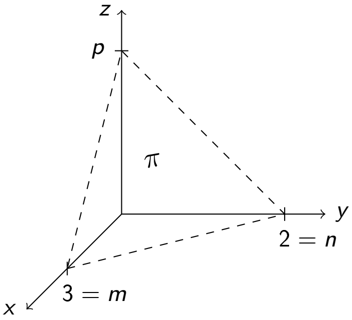
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned}
\pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\
&\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \\
&\Leftrightarrow \quad \tfrac{1}{3} \cdot 2 + \tfrac{1}{2} \cdot (-1) + \tfrac{1}{p} \cdot 1 = 0 \\
&\Leftrightarrow \quad \tfrac{1}{p} = -\tfrac{1}{6}
\end{aligned}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\dfrac{x}{m} + \dfrac{y}{n} + \dfrac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \dots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

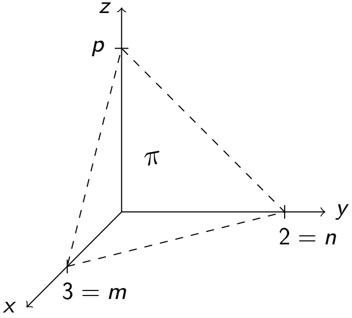
za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned}
\pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\
&\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \\
&\Leftrightarrow \quad \tfrac{1}{3} \cdot 2 + \tfrac{1}{2} \cdot (-1) + \tfrac{1}{p} \cdot 1 = 0 \\
&\Leftrightarrow \quad \tfrac{1}{p} = -\tfrac{1}{6} \quad \Leftrightarrow \quad p = -6.
\end{aligned}$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\frac{x}{m} + \frac{y}{n} + \frac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned}
\pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\
&\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \\
&\Leftrightarrow \quad \tfrac{1}{3} \cdot 2 + \tfrac{1}{2} \cdot (-1) + \tfrac{1}{p} \cdot 1 = 0 \\
&\Leftrightarrow \quad \tfrac{1}{p} = -\tfrac{1}{6} \quad \Leftrightarrow \quad p = -6.
\end{aligned}$$

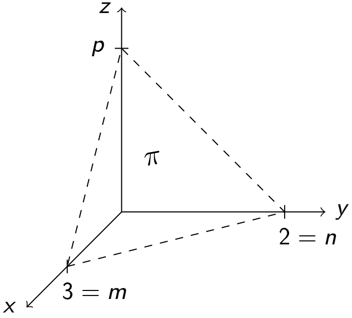
Dakle,

$$\pi \ldots \frac{x}{3} + \frac{y}{2} + \frac{z}{-6} = 1$$

Zadatak 70

Odredite jednadžbu ravnine π paralelne s vektorom $\vec{v} = [2, -1, 1]$ koja x-os siječe u točki $(3, 0, 0)$, a y-os u točki $(0, 2, 0)$.

Rješenje.



Ravnina π ima segmentni oblik jednadžbe $\frac{x}{m} + \frac{y}{n} + \frac{z}{p} = 1$, gdje je $m = 3$ i $n = 2$, dakle

$$\pi \ldots \ \frac{x}{3} + \frac{y}{2} + \frac{z}{p} = 1$$

za neki $p \in \mathbb{R} \setminus \{0\}$. Odredimo p. Imamo

$$\begin{aligned}
\pi \parallel \vec{v} \quad &\Leftrightarrow \quad \vec{n}_\pi \perp \vec{v} \quad \Leftrightarrow \quad \vec{n}_\pi \cdot \vec{v} = 0 \\
&\Leftrightarrow \quad \left[\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{p}\right] \cdot [2, -1, 1] = 0 \\
&\Leftrightarrow \quad \tfrac{1}{3} \cdot 2 + \tfrac{1}{2} \cdot (-1) + \tfrac{1}{p} \cdot 1 = 0 \\
&\Leftrightarrow \quad \tfrac{1}{p} = -\tfrac{1}{6} \quad \Leftrightarrow \quad p = -6.
\end{aligned}$$

Dakle, $\quad \pi \ldots \frac{x}{3} + \frac{y}{2} + \frac{z}{-6} = 1, \qquad$ tj. (množenjem sa 6) $\qquad \pi \ldots \ 2x + 3y - z = 6.$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Rješenje. Imamo

$$p \parallel \pi_1, \pi_2 \quad \Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2}$$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots \; 3x + 12y - 3z = 5$ i $\pi_2 \ldots \; 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots \; \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots \; \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Rješenje. Imamo

$$
\begin{aligned}
p \parallel \pi_1, \pi_2 \quad &\Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2} \\
&\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2}
\end{aligned}
$$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x+12y-3z=5$ i $\pi_2 \ldots\ 3x-4y+9z=7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2}=\frac{y-3}{-4}=\frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2}=\frac{y+1}{3}=\frac{z-2}{4}$.

Rješenje. Imamo

$$
\begin{aligned}
p \parallel \pi_1, \pi_2 \quad &\Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2} \\
&\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 12 & -3 \\ 3 & -4 & 9 \end{vmatrix}
\end{aligned}
$$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Rješenje. Imamo

$$\begin{aligned} p \parallel \pi_1, \pi_2 \quad &\Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2} \\ &\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 12 & -3 \\ 3 & -4 & 9 \end{vmatrix} = [96, -36, -48] \end{aligned}$$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Rješenje. Imamo

$$\begin{aligned} p \parallel \pi_1, \pi_2 \quad &\Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2} \\ &\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 12 & -3 \\ 3 & -4 & 9 \end{vmatrix} = [96, -36, -48] \end{aligned}$$

pa možemo staviti npr. $\vec{s}_p = \frac{1}{12}[96, -36, -48]$

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

Rješenje. Imamo

$$\begin{aligned} p \parallel \pi_1, \pi_2 \quad &\Rightarrow \quad \vec{s}_p \perp \vec{n}_{\pi_1}, \vec{n}_{\pi_2} \\ &\Rightarrow \quad \vec{s}_p \parallel \vec{n}_{\pi_1} \times \vec{n}_{\pi_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 12 & -3 \\ 3 & -4 & 9 \end{vmatrix} = [96, -36, -48] \end{aligned}$$

pa možemo staviti npr. $\vec{s}_p = \frac{1}{12}[96, -36, -48] = [8, -3, -4]$.

Zadatak 71

Odredite jednadžbu pravca p koji je paralelan ravninama $\pi_1 \ldots\ 3x + 12y - 3z = 5$ i $\pi_2 \ldots\ 3x - 4y + 9z = 7$ i siječe pravce $q_1 \ldots\ \frac{x+5}{-2} = \frac{y-3}{4} = \frac{z+1}{-3}$ i $q_2 \ldots\ \frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$.

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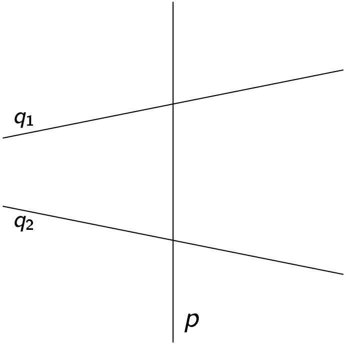

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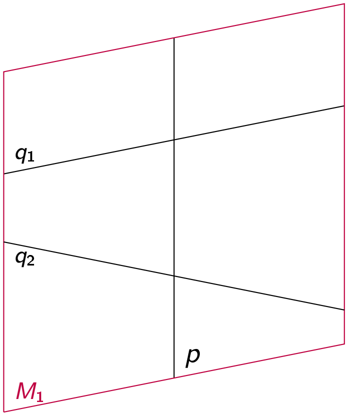

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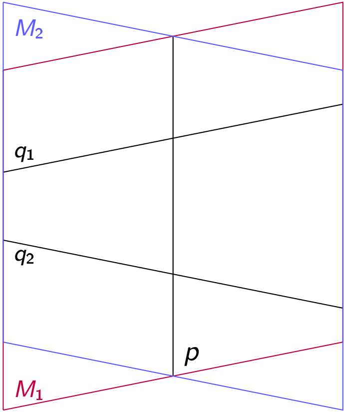

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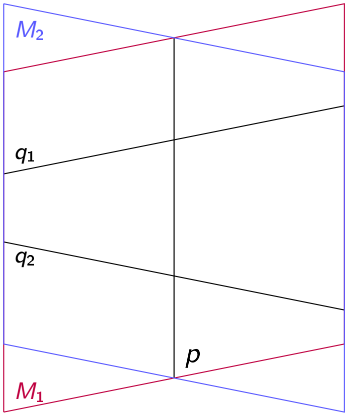

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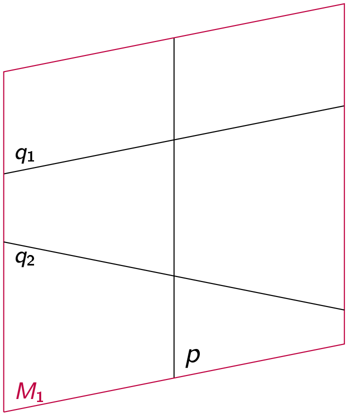

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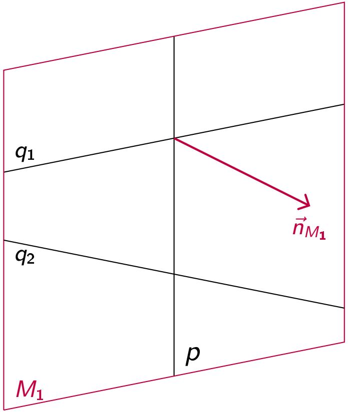

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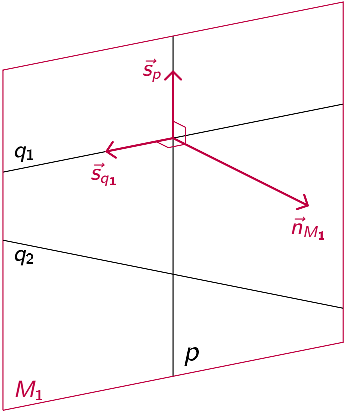

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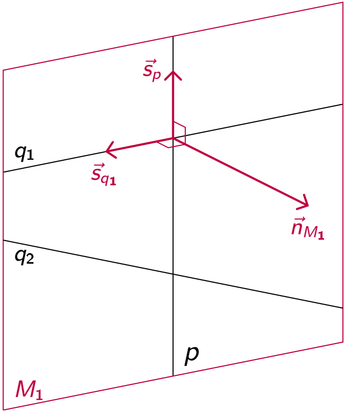

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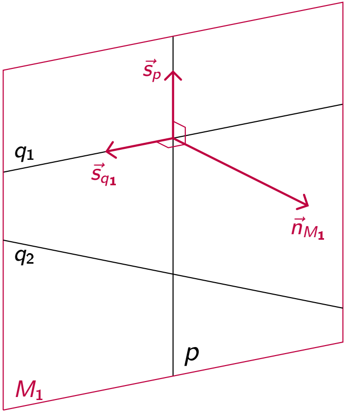

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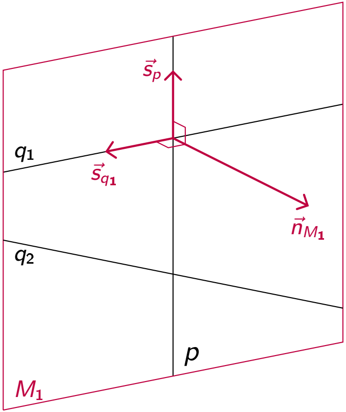

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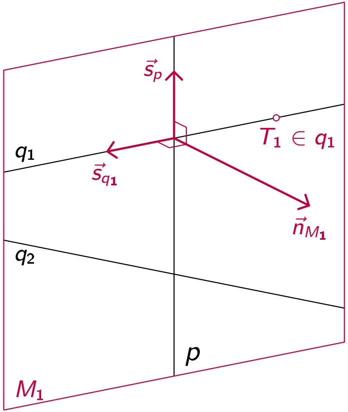

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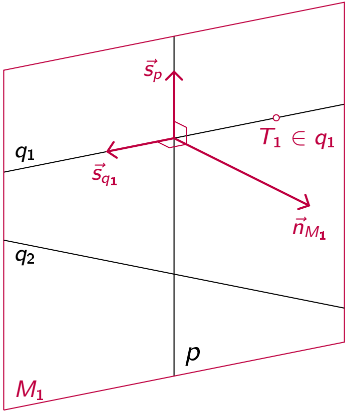

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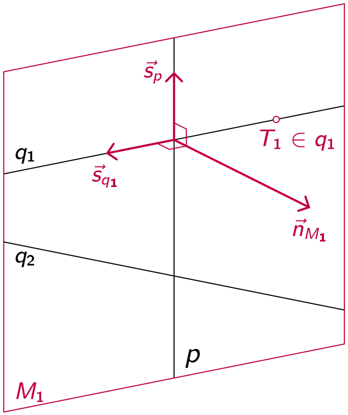

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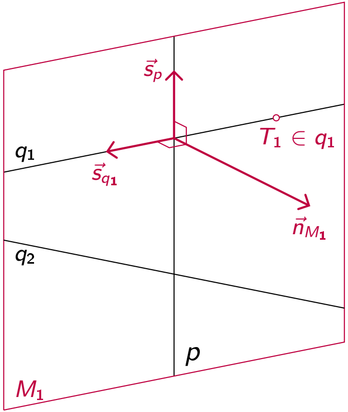

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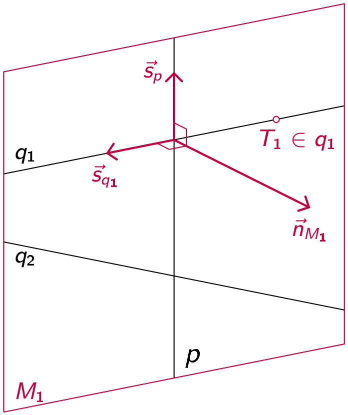

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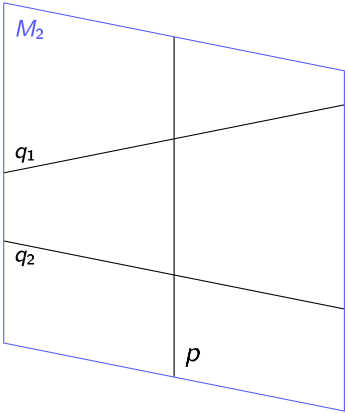

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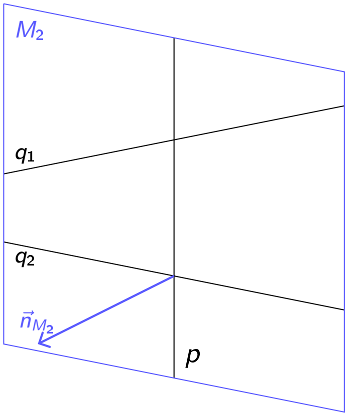

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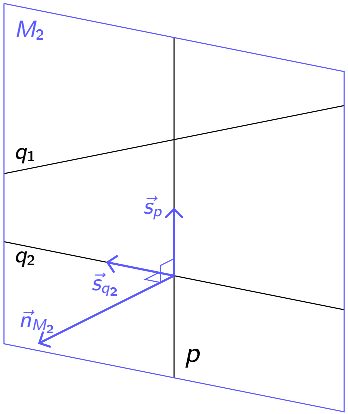

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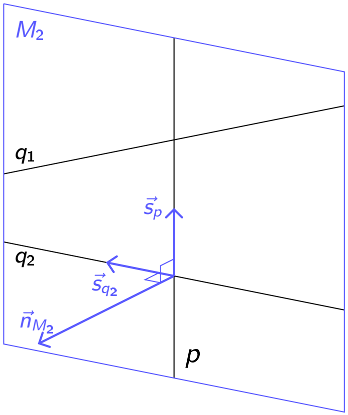

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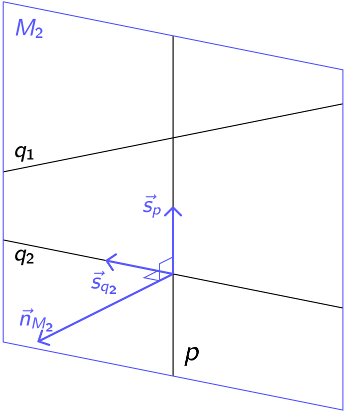

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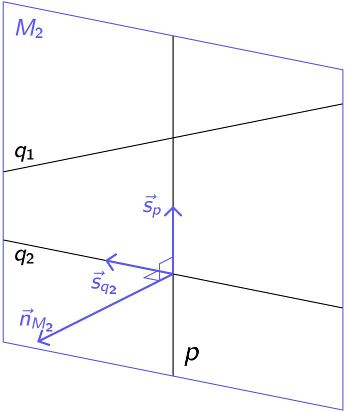

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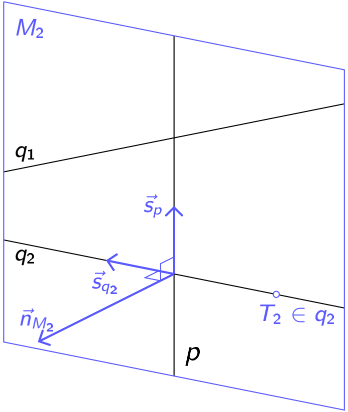

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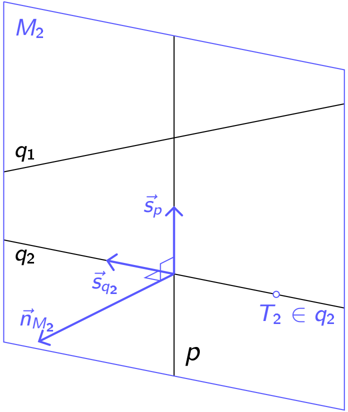

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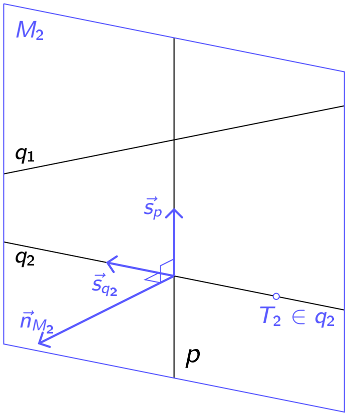

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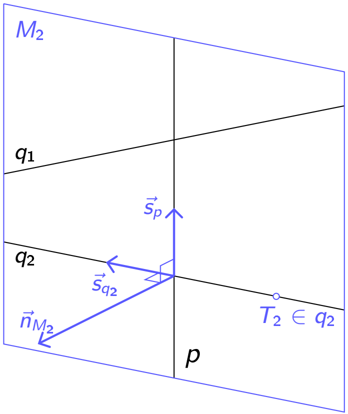

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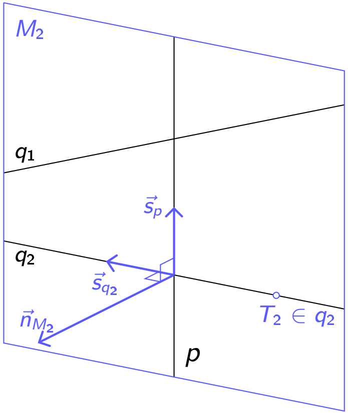

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$$4y - 3z = -10.$$

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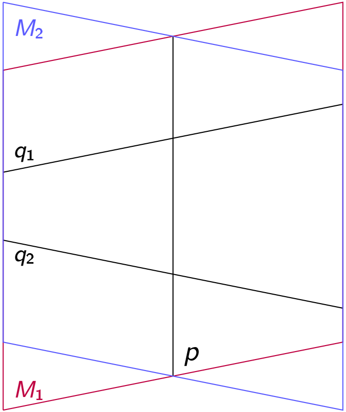
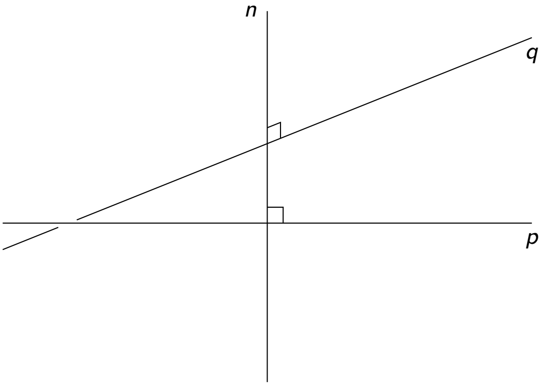


Dakle, implicitna jednadžba pravca p dana je sa

$$p \ldots \begin{cases} M_1 \ldots\ 25x + 32y + 26z = -55 \\ M_2 \ldots\ 4y - 3z = -10. \end{cases}$$

Zajednička normala mimoilaznih pravaca

Definicija. **Zajednička normala** mimoilaznih pravaca p i q jest pravac n koji siječe pravce p i q i okomit je na svaki od njih.



Napomena. Zajednička normala dvaju mimoilaznih pravaca uvijek postoji i jedinstvena je.

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Odredite kanonski oblik jednadžbe zajedničke normale n pravaca $p \ldots \ \frac{x}{1} = \frac{y}{1} = \frac{z}{0}$ i $q \ldots \ \frac{x-1}{0} = \frac{y-2}{1} = \frac{z-3}{0}$.

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Rješenje. Imamo

$$n \perp p, q$$

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$$\begin{aligned} n \perp p, q \quad &\Rightarrow \quad \vec{s}_n \perp \vec{s}_p, \vec{s}_q \\ &\Rightarrow \quad \vec{s}_n \parallel \vec{s}_p \times \vec{s}_q \end{aligned}$$

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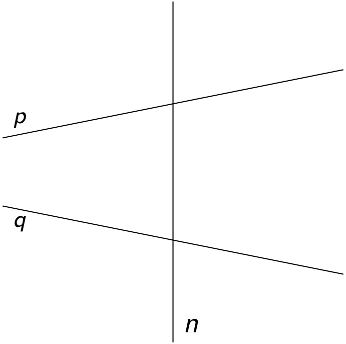

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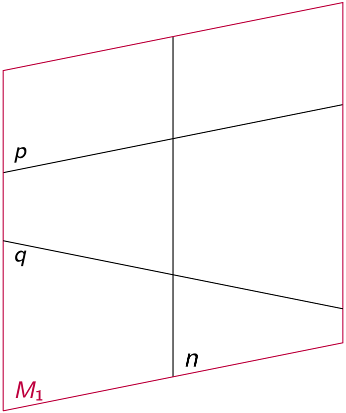

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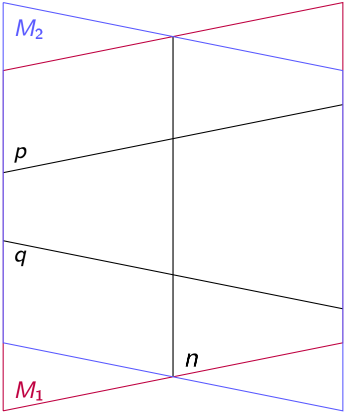

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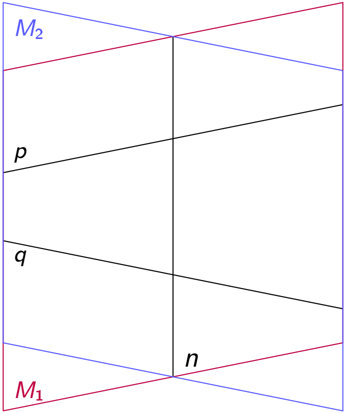

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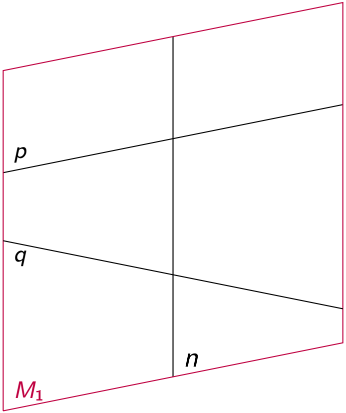

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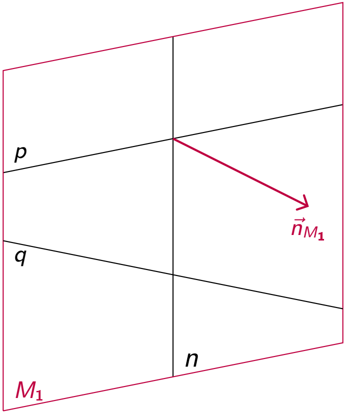

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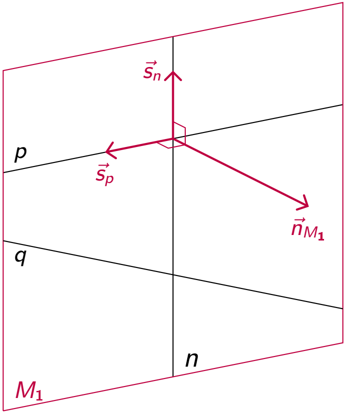

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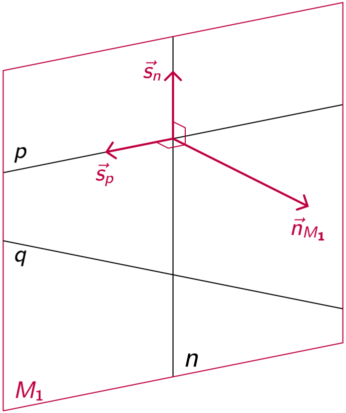

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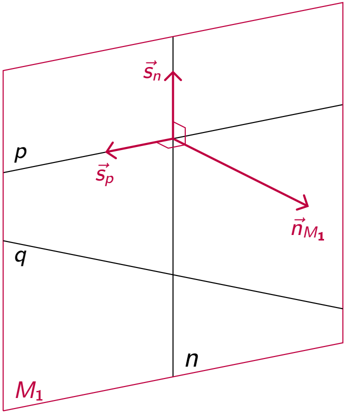

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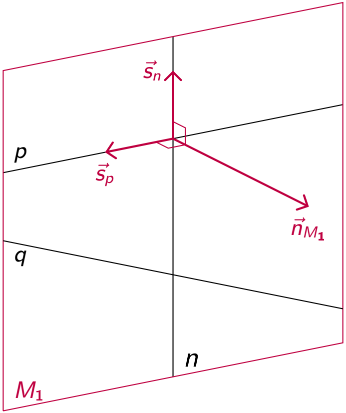

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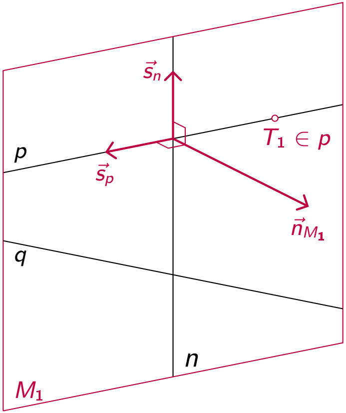

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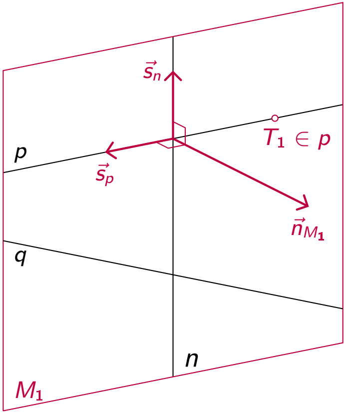

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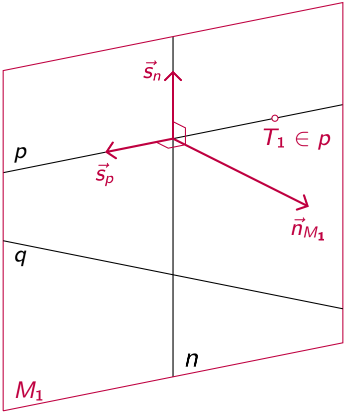

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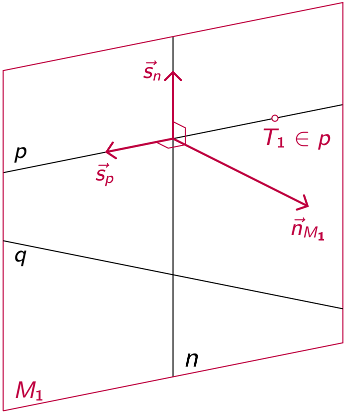

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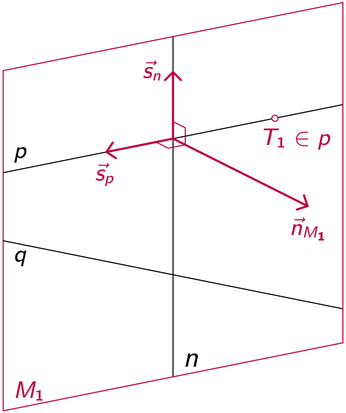

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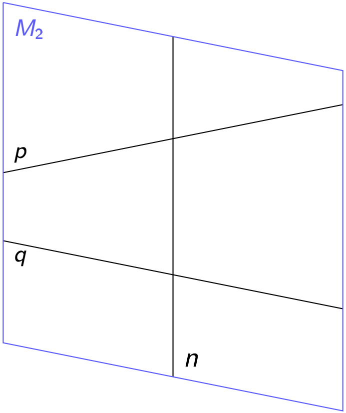

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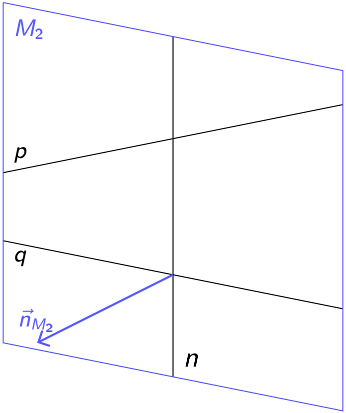

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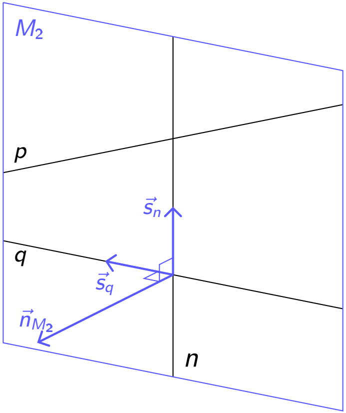

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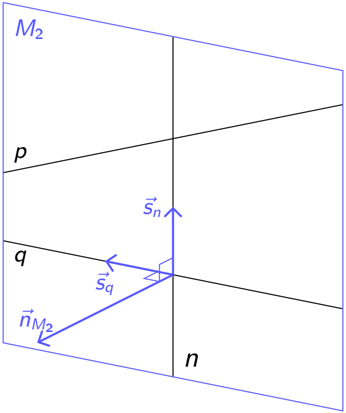

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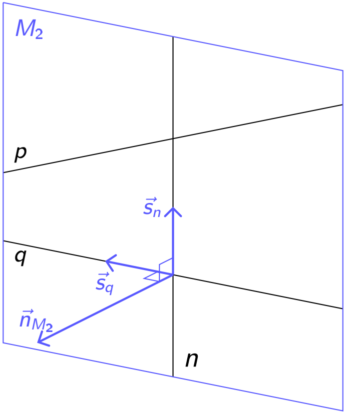

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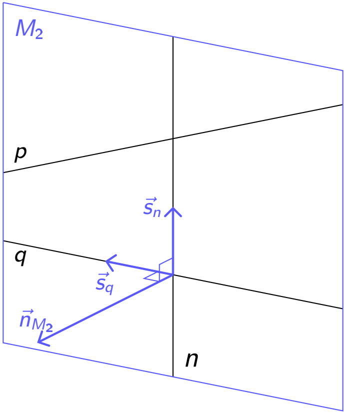

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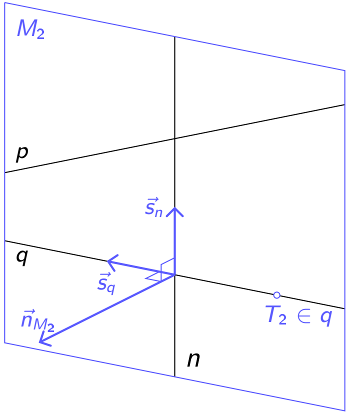

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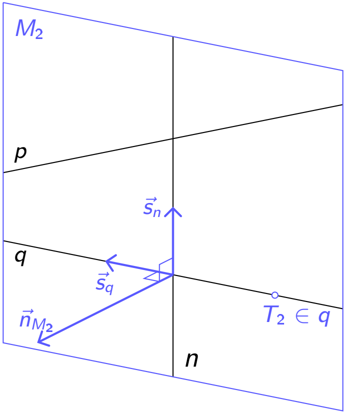

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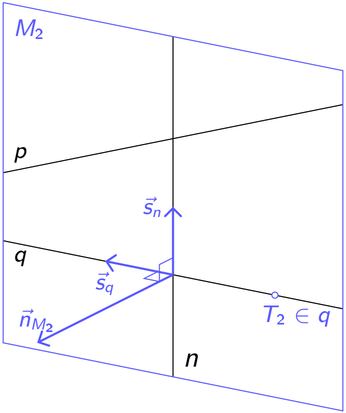

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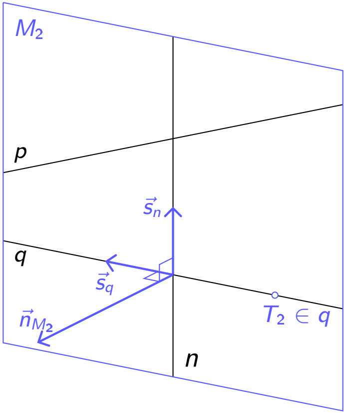

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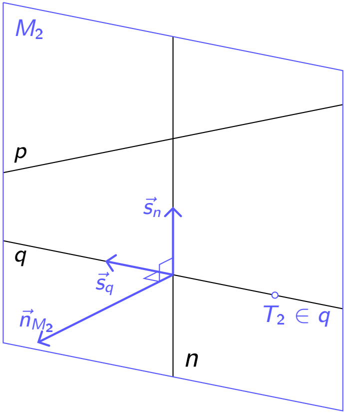

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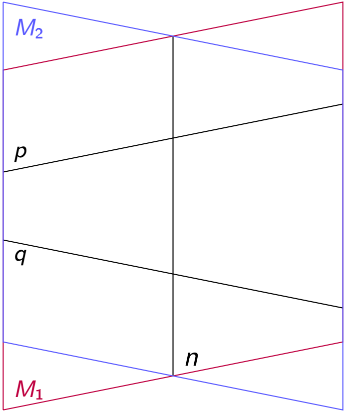

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Dakle, implicitna jednadžba pravca n dana je sa

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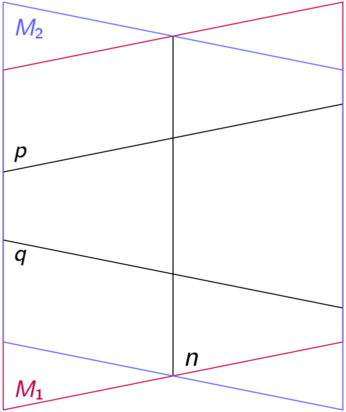

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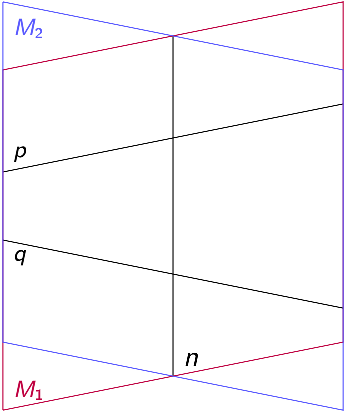

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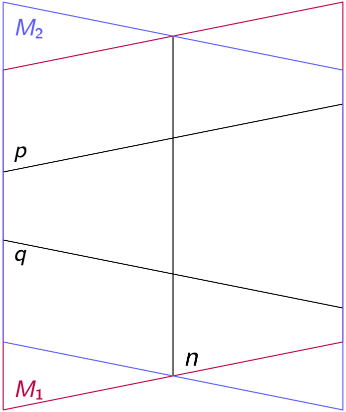
$$n \ldots \begin{cases} y = x \\ x = 1 \end{cases}$$

Zadatak 72

Odredite kanonski oblik jednadžbe zajedničke normale n pravaca $p \ldots \frac{x}{1} = \frac{y}{1} = \frac{z}{0}$ i $q \ldots \frac{x-1}{0} = \frac{y-2}{1} = \frac{z-3}{0}$.

Rješenje. $\rightsquigarrow \quad \vec{s}_n = [0, 0, 1]$.



Ideja. Pravac n je presjek ravnina $M_1 \supseteq n, p$ i $M_2 \supseteq n, q$.

Dakle, implicitna jednadžba pravca n dana je sa

$$n \ldots \begin{cases} M_1 \ldots \ x - y = 0 \\ M_2 \ldots \ x = 1. \end{cases}$$

Odredimo njegovu parametarsku jednadžbu. Imamo

$$n \ldots \begin{cases} y = x \\ x = 1 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x = 1 \\ \\ \end{cases}$$

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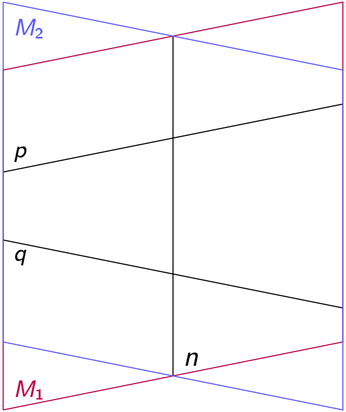

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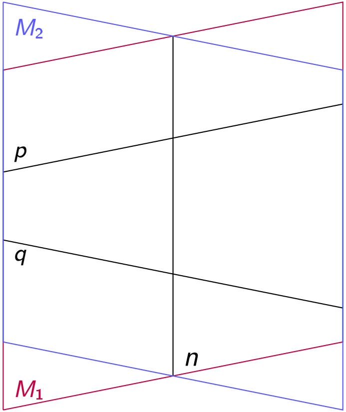

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Zadatak 72

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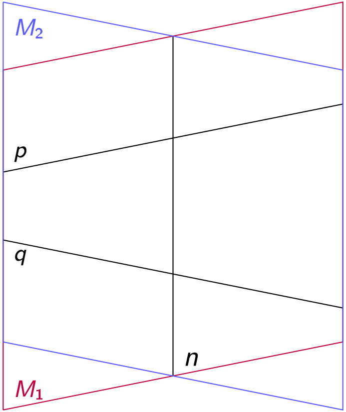

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Dakle, kanonski oblik jednadžbe pravca n je

$$n \ldots \frac{x-1}{0} = \frac{y-1}{0} = \frac{z}{1}.$$